NMSU MATH PROBLEM OF THE WEEK

Solution to Problem 2

Spring 2026

Problem 2

A large $5 \times 5 \times 5$ cube is made up of 125 smaller $1 \times 1 \times 1$ unit cubes. The entire exterior of the large cube is painted NMSU Crimson. Once the paint is dry, the large cube is disassembled back into the 125 small unit cubes. How many of the small cubes have exactly two faces painted Crimson? How many of the small cubes have no paint on them at all? Justify your answer.

Solution. We observe the following:

- The large cube has 8 small corner cubes, so **8** small cubes have exactly three faces painted Crimson.
- Each face of the large cube has 9 interior small cubes, so on each face there are 9 small cubes that have exactly one face painted Crimson. Since the large cube has 6 faces, there are $9 \cdot 6 = \mathbf{54}$ small cubes that have exactly one face painted Crimson.
- In addition, there are **27** interior small cubes that have no paint at all.
- Adding up the above gives us $8 + 54 + 27 = 89$ small cubes, so the remaining $125 - 89 = \mathbf{36}$ small cubes have exactly two faces painted Crimson.

Thus, there are **36** small cubes that have exactly two faces painted Crimson and **27** small cubes that have no paint at all.